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Backcasting and forecasting time series using detrended cross-correlation analysis

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ABSTRACT

In this paper we addressed the problem of backcasting and forecasting non-stationary time series. Given that traditional backcasting are based on the classical cross-correlation function assuming two stationary time series, assumptions based on two non-stationary time series need a new approach. Thus, Detrended Cross-Correlation Analysis (DCCA) provides a fanned estimator for a correlation between two non-stationary time series, providing then the stationary time series. Therefore, we developed a backcaster estimator and a predictor for the obtained stationary time series. The practical example of a Chilean cement production time series (1991–2015) illustrates our results.

1. Introduction

Detrended cross correlation analysis (DCCA; [1]) has been widely used to identify and characterize correlated data in several scientific fields: transport [2], economy [3,4], geology [5], meteorology [6], among others. The key idea of DCCA is to characterize different fluctuation properties presented in time series, to then separate actual fluctuations from contributions due to local, global and periodic trends that persist over larger scales [7]. DCCA provides a fanned estimator for a correlation between two non-stationary time series, providing then stationary time series.

Different methodologies have been proposed depending on the time series' structure. Early work by Kantelhardt et al. [8] presents a multifractal version of detrended fluctuation analysis (DFA) of non-stationary time series. In Podobnik and Stanley [1] and Podobnik et al. [9] a DCCA cross-correlation coefficient is developed for fractionally autoregressive integrated moving average (ARFIMA; see e.g. [10]) processes with periodic trends. Theoretical approaches were addressed by Zebende et al. [11] to establish a relationship between the long-range auto-correlation exponent and the long-range cross-correlation exponent.

The theorem proposed by Idrovo-Aguirre and Contreras-Reyes [12] assumes weak stationarity in time series to establish a strict and a cointegration-based backcaster. In practice, given that time series are generally non-stationary, the classical cross-correlation function (CCF) and Ljung–Box test for white noise [13] are based on wrong assumptions. Moreover, in Idrovo-Aguirre and Contreras-Reyes [12] the strictly linear-backward and cointegration-based backward projection methods are used to determine new series of splicing cement manufacturing in the unobserved period (1991–2015), using as backward projection instrument the old dispatch indicator, where both time series share a long-term relationship. Particularly, strict backcasting supposes the observations of the backcasting variable and splice instruments come from the same data generating process, and therefore the residual structure of the relationship between variables is white

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noise. This paper is motivated by the need to have a backcaster estimator and a predictor assuming that two time series are non-stationary.

This paper proceeds as follows: after this introduction, DCCA is applied to backcasting non-stationary time-series and its implications regarding predictive analysis in Section 2. Section 3 presents some results of the application of proposed methodologies to Chilean cement production time series (1991–2015). Conclusions and the discussion then appear in Section 4. The proposed methodologies were implemented in R software [14].

2. Detrended cross-correlation analysis

In this section, we implement the proposed DCCA cross-correlation among two non-stationary time series [1,15]. Consider two non-stationary time series, $\{x_t\}$ and $\{y_t\}$, $t = 1, \dots, N$. Now, consider two integrated time series,

$$R_k = \sum_{t=1}^{k}(x_t - \bar{x}), \quad \bar{x} = \frac{1}{N}\sum_{t=1}^{N} x_t,$$
$$S_k = \sum_{t=1}^{k}(y_t - \bar{y}), \quad \bar{y} = \frac{1}{N}\sum_{t=1}^{N} y_t, \quad k = 1, \dots, N; \tag{2.1}$$

i.e., they are obtained by the cumulative sum (CUSUM), a classic procedure that produces the CUSUM of deviations, x_t and y_t, from a target specification, $\bar{x}$ and $\bar{y}$, respectively [16]. The distribution of resulting CUSUM series has a zero mean. In CUSUM space, positive values indicate a deviation of time series above the average, while negative values indicate a deviation below the average. Breakpoints where CUSUM trends transition from negative to positive slopes (or vice versa), represent a shift in the sample from lower to higher than average (or vice versa) [17].

Next, both time series are divided into $N-n$ overlapping boxes, each containing $n+1$ observations. Then, the covariance of the residuals in each box of size $n + 1$ is

$$f^2_{DCCA}(n, t, R, S) = \frac{1}{n+1}\sum_{k=t}^{t+n}(R_k - \overline{R_k})(S_k - \overline{S_k}), \tag{2.2}$$

where $\overline{R_k}$ and $\overline{S_k}$ ($t \le k \le t + n$) are local averages for detrended time series. To implement the DCCA cross-correlation coefficient, we need to average time-lagged covariance over all overlapping $N - n - |\tau|$ boxes, $\tau \in \mathbb{N}$, where τ is the time lag [15]. Considering the covariance of Eq. (2.2), the time-lagged covariance in each box depends on τ and is calculated as

$$f^2_{DCCA}(n, t, \tau, R, S) = \frac{1}{n+1}\sum_{k=t}^{t+n}(R_k - \overline{R_k})(S_{k+\tau} - \overline{S_{k+\tau}}). \tag{2.3}$$

Depending on the sign of τ, the DCCA covariance function is

$$F^2_{DCCA}(n, \tau, R, S) = \begin{cases} \frac{1}{N-n-\tau}\sum_{t=1}^{N-n-\tau} f^2_{DCCA}(n, t, \tau, R, S), & \tau \ge 0; \\ \frac{1}{N-n-\tau}\sum_{t=-\tau+1}^{N-n} f^2_{DCCA}(n, t, \tau, R, S), & \tau < 0, \end{cases} \tag{2.4}$$

where $f^2_{DCCA}(n, t, \tau, R, S)$ is given in Eq. (2.3). Finally, the DCCA cross-correlation coefficient is

$$\rho(n, \tau, R, S) = \frac{F^2_{DCCA}(n, \tau, R, S)}{F_{DCCA}(n, \tau, R, R)F_{DCCA}(n, \tau, S, S)}, \tag{2.5}$$

and the DCCA cross-correlation coefficients of each R_k and S_k time series are

$$\rho(n, \tau, R, R) = \frac{F^2_{DCCA}(n, \tau, R, R)}{F_{DCCA}(n, 0, R, R)} \quad \text{and} \quad \rho(n, \tau, S, S) = \frac{F^2_{DCCA}(n, \tau, S, S)}{F_{DCCA}(n, 0, S, S)}, \tag{2.6}$$

respectively.

The value of $\rho(n, \tau, R, S)$ ranges between -1 and 1. A value of $\rho(n, \tau, R, S) = 0$ means there is no cross-correlation, and it splits the level of cross-correlation between positive and the negative case [2].

To investigate the cross-correlations between R and S, we apply a cross-correlation statistic of Ljung–Box type based on chi-squared distribution (see [13]) and proposed by Podobnik et al. [9,18]. The statistic $\mathcal{Q}$ is defined as

$$\mathcal{Q}(k) = n(n+2)\sum_{i=1}^{k}\frac{\rho(n, i, R, S)^2}{n-i}, \tag{2.7}$$

and is approximately χ^2_k distributed with k degrees of freedom. The statistical test has good agreement with the χ^2_k distribution if there are no cross-correlations between the time series [15]. If the statistic given in (2.7) exceeds the critical value of the χ_k^2 distribution, the cross correlations are significant at a special significance level α (typically used $\alpha = 0.05$ for a 95% confidence level). This means, if the associated p-value is equal or less than confidence probability:

$$\mathbb{P}(\chi_k^2 > \mathcal{Q}(k)) \leq \alpha, \quad 0 < \alpha < 1, \tag{2.8}$$

the cross-correlation is statistically significant.

3. Backcasting and forecasting detrended time series

The theorem proposed by Idrovo-Aguirre and Contreras-Reyes [12] states that two time series are generated by the same process if their difference presents the characteristics of a white noise process, while matching their simple auto-correlation functions with a CCF. In the following theorem, we adopted the stationary R_k and S_k time series obtained via the CUSUM approach to the theorem proposed by Idrovo-Aguirre and Contreras-Reyes [12].

Theorem 3.1. *Let $\{R_k\}$ and $\{S_k\}$, $k \in \mathbb{N}$, be two weak stationary processes with means $\langle R_k\rangle = \mu$ and $\langle S_k\rangle = \xi$ and detrended auto-covariances given by $f_{DCCA}^2(n,t,R,R)$ and $f_{DCCA}^2(n,t,S,S)$, respectively; under the conditions*

$$\sum_{j=0}^{\infty} |f_{DCCA}^2(n,j,R,R)| < \infty \quad \textit{and} \quad \sum_{j=0}^{\infty} |f_{DCCA}^2(n,j,S,S)| < \infty.$$

Considering the linear relation $R_k = S_k + \varepsilon_k$, $\varepsilon_k \sim WN(0,\sigma^2)$ (white noise with mean zero and variance σ^2), $R_k \in \mathcal{F}_{k+1}$, $\mathcal{F}_{k+1} \equiv \overline{sp}\{S_s : s < k+1,\ s \in \mathbb{N}\}$, where $\overline{sp}$ is the generated space or span that includes its boundary. From (2.5) and (2.6)*, it follows that*

$$\rho(n,\tau,R,R) = \rho(n,\tau,S,S) = \rho(n,\tau,R,S), \quad \forall \tau \in \mathbb{N}.$$

Proof of Theorem 3.1. Given the linear relationship $R_k = S_k + \varepsilon_k$ between $\{R_k\}$ and $\{S_k\}$, we have their respective detrended covariance functions $f_{DCCA}^2(n,j,R,R)$ and $f_{DCCA}^2(n,j,S,S)$. Furthermore, $\langle R_k\rangle = \langle S_k\rangle$ since applying the classical assumption that $\langle \varepsilon_k\rangle = 0$. Under the assumption $R_k \in \mathcal{F}_{k+1}$, we have $f_{DCCA}^2(n,j,R,R) = f_{DCCA}^2(n,j,S,S)$, and thus $\rho(n,j,R,R) = \rho(n,j,S,S)$, $j = 0,1,\ldots$. Then, since

$$\frac{f_{DCCA}^2(n,j,R,R)}{f_{DCCA}^2(n,0,R,R)} = \frac{f_{DCCA}^2(n,j,R,S)}{\sqrt{f_{DCCA}^2(n,0,R,R)^2}},$$

we have $\rho(n,j,R,R) = \rho(n,j,R,S)$, $j = 0,1,\ldots$. Since $\rho(n,j,R,S) < \infty$, we also have

$$\sum_{j=0}^{\infty} \rho(n,j,R,R) = \sum_{j=0}^{\infty} \rho(n,j,R,S) < \infty. \quad \square$$

Theorem 1 proposed by Idrovo-Aguirre and Contreras-Reyes [12] is used here assuming sample covariance functions, instead of theoretical covariance functions. In addition, Theorem 3.1 is equivalent to Theorem 1 of Idrovo-Aguirre and Contreras-Reyes [12] assuming weak stationary time series. In the case $|\rho(n,\tau,R,S)| \approx 1$, we can backcast the new time series R_k based on rates of implied variation of S_k used as a splice instrument. Therefore, the backcasted detrended time series is

$$\widehat{R}_{q-(j+1)} = \frac{R_{q-j}S_{q-(j+1)}}{S_{q-j}}, \quad j = 0,1,\ldots,q-1; \tag{3.9}$$

where q ($q < N$) is the time step in which the variable $\{R_k\}$ to be backcasted is observable. In addition, $\{S_k\}$ is used for the backcasting, given that it is highly correlated with $\{R_k\}$ and by its observable pattern of the past. In this case, $k = 1,\ldots,n+1$, corresponds to a set of observations of the first box with $N-n$ total possible divisions. This is given by the relationship between $\{R_k\}$ and $\{S_k\}$ obtained in the first box. The first box represents the closest box (among all boxes) that covers the backcasting, i.e., $j = 0,\ldots,q-1$.

From Eq. (3.9) and in line with assumptions of Theorem 3.1, we can compute the variance at each j, as

$$\langle \widehat{R}_{k-(j+1)}, \widehat{R}_{k-(j+1)}\rangle = \left(\frac{S_{k-(j+1)}}{S_{k-j}}\right)^2 \langle \varepsilon_{k-j}, \varepsilon_{k-j}\rangle = \left(\sigma_b \frac{S_{k-(j+1)}}{S_{k-j}}\right)^2, \tag{3.10}$$

i.e., $\sigma^2 = \sigma_b^2$ is related to backcasted R_k. Then, it is possible to obtain the respective $100(1-\alpha)$% confidence bands of the backcasted time series as

$$\left[\widehat{R}_{q-(j+1)} \pm z_{1-\alpha}\left(\widehat{\sigma}_b \frac{S_{k-(j+1)}}{S_{k-j}}\right)\right], \tag{3.11}$$

where $z_{1-\alpha}$ denotes the standardized normal quantile related to a significance level of α, and $\widehat{\sigma}_b$ is the sample standard deviation of the random vector $\{\widehat{R}_{q-1} - S_{q-1},\ldots,\widehat{R}_0 - S_0\}$ as an unbiased estimator of σ_b (see Theorem 3.1).

In some particular cases, it could be necessary to complete the detrended time series at "future" time steps, $j = q, q+1, \ldots, N-1$. Thus, Theorem 3.1 provides a predictor for detrended time series based on a known variable S_k,

$$\widehat{R}_{j+1} = \frac{R_j S_{j+1}}{S_j}, \quad j = q, q+1, \ldots, N-1; \tag{3.12}$$

and its respective variance,

$$\langle \widehat{R}_{j+1}, \widehat{R}_{j+1} \rangle = \left(\frac{S_{j+1}}{S_j} \right)^2 \langle \varepsilon_j, \varepsilon_j \rangle = \left(\sigma_f \frac{S_{j+1}}{S_j} \right)^2, \tag{3.13}$$

i.e., $\sigma^2 = \sigma_f^2$ is related to forecasted R_k. Then, it is possible to obtain the respective $100(1-\alpha)\%$ confidence bands of the forecasted time series as

$$\left[\widehat{R}_{j+1} \pm z_{1-\alpha} \left(\widehat{\sigma_f} \frac{S_{j+1}}{S_j} \right) \right], \tag{3.14}$$

where $\widehat{\sigma_f}$ is the sample standard deviation of the random vector $\{\widehat{R}_{q+1} - S_{q+1}, \ldots, \widehat{R}_N - S_N\}$ as an unbiased estimator of σ_f (see Theorem 3.1).

Considering an specific time step q^*, $q < q^* < N$, chosen from the minimum value of the available detrended time series R_k, i.e.,

$$q^* = \arg \min_{q \le k \le N} \{R_k\}; \tag{3.15}$$

it is possible to initialize the backcasting and forecasting procedures described in Eqs. (3.9) and (3.12), respectively. The rule proposed by Page [16] suggests that, if $R_k - R_{q^*} \ge h$, $q \le k \le N$, $h > 0$; then a change direction at time k is triggered, where h is a threshold considering a significant proportion of the total range of R_k (see Figure 1 of [16,17]). Page's criterion was designed as a process inspection scheme to detect deviations in average in only one direction (one-sided).

4. Application to cement's production

The application of the strict linear backward projection method (RLE) requires both series (cement production and sales) to be derived from the same process or function data generator probability density. It should be noted that the application of this technique considers the methodological assumption that cement can be treated as a non-storable input, according to the analysis in the Introduction. To validate the relative coincidence between levels of cement and tons shipped of the same input, at least during the period shared by both series (2009–2015), Idrovo-Aguirre and Contreras-Reyes [12] statistically tested the hypothesis that the difference between cement supply and demand is white noise. In this case, it is possible to apply the strict linear backward projection technique. However, it was also found that both time series were not stationary on original, logarithmic, and seasonal levels using Augmented Dickey–Fuller unit root test [19]. In addition, the Ljung–Box test indicates that residues of the relationship between integrated cement production and sales time series are not white noise.

In Idrovo-Aguirre and Contreras-Reyes [12], the backcasted cement production is performed considering that both time series (x_t for production and y_t for sales) are non-stationary. Thus, having verified the existence of cointegration, the backward projection of cement production conditioned to the observed dispatches as splice variable is given by

$$\widehat{x}_{q-(j+1)} = -\left(\frac{\widehat{\alpha}}{\widehat{\beta}} \right) y_{q-(j+1)}, \quad j = 0, 1, \ldots, q-1; \tag{4.16}$$

where q is the step in which the variable x_k to be backcasted is observable (i.e., $q = 216$), and $\widehat{\alpha}$ and $\widehat{\beta}$ are obtained by the cointegration method in which $\widehat{\alpha}/\widehat{\beta} = -0.641$ (t-value=10.917; s.e=0.059). Therefore, backcasted cement production based on Eq. (4.16) and demand fluctuations are presented in Fig. 1. From 2009–2015, changes in the cement production stemmed from the international financial crisis of 2009, where cycle effects of mining investment are evident in the evolution of sectoral investment [12].

4.1. Detrended cross-correlation analysis

Panel (a) of Fig. 2 shows the detrended monthly time series of cement (backcasted) production and demand fluctuations, following Eq. (2.1). Clearly, there are extended, contiguous periods where cement production and demand time series are higher or lower than their respective averages. Consistently negative slopes indicate periods when most values are below the average, while positive slopes indicate periods when most values are above the average. Considering a *R/S* Hurst estimation (see e.g. [20]) [implemented in the `hurstexp` function of `pracma` package [21] of `R` software [14]], the estimated Hurst exponent was $H = 0.753$, indicating long-range auto-correlation with persistent behavior ($0.5 < H < 1.0$). In panel (b), we find that the coefficient ρ_{DCCA} between both time series is a persistent cross correlation at short time scales (entire evaluated period). Besides, the Ljung–Box test [Eqs. (2.7) and (2.8)] gives $\mathbb{P}(\chi_k^2 > \mathcal{Q}(k)) \le 0.05$, $k = 1, \ldots, 60$. Thus, these DCCA coefficients are statistically significant at 95% confidence level; corroborating the assumption of Theorem 3.1.

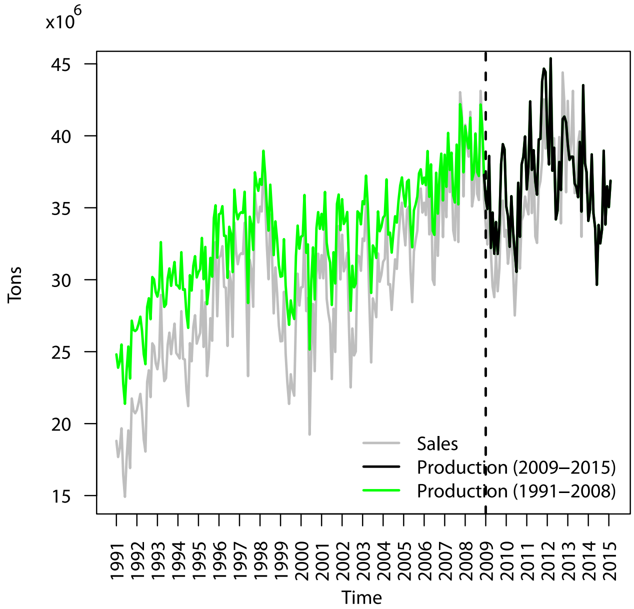

Fig. 1. Monthly time series of cement production (2009–2015), backcasted cement production (1991–2008) using integration method of Eq. (4.16), and sales fluctuations (1991–2015). The highlighted time step (dashed line) corresponds to $q = 216$ (2009/01).

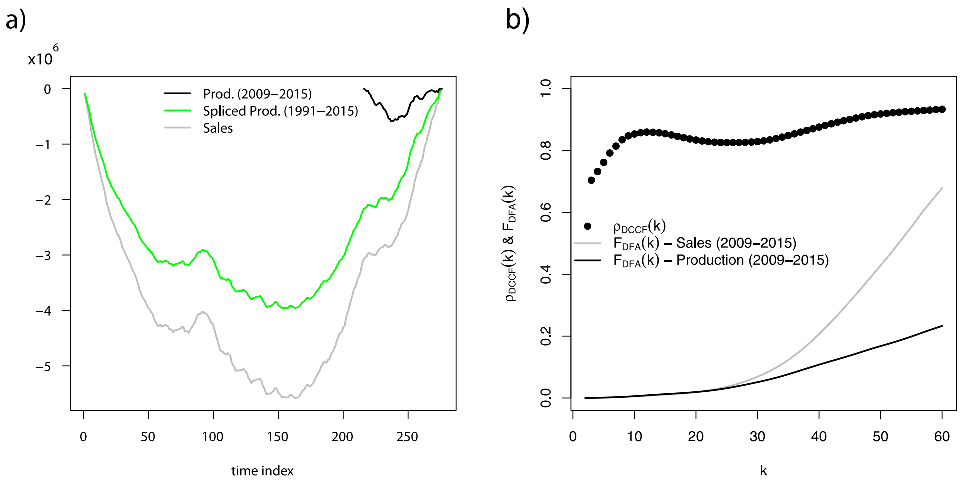

Fig. 2. (a) Backcasted monthly time series of cement production and demand fluctuations. The spliced production time series (1991–2015) is based on Eq. (4.16) of Idrovo-Aguirre and Contreras-Reyes [12]. (b) Cross correlation coefficient on different time scales k between log-transformed cement production and monthly demand time series with their respective DFA coefficients.

4.2. Backcasting and forecasting detrended cement production

For available data (2009–2015), we observed in panel (a) of Fig. 3 a minimum value trigged at $q^* = 237$ for the detrended production (DP) time series. Page's criterion based on q^* is considered here because the DP time series is close to zero at $q = 237$ and the 1991–2015 time series is close to zero at $k = 1$. Thus, detrended production (BDP-q^*) is backcasted from 1991 to 2008 for q^* using Eq. (3.9).

We observed the clear influence of detrended sales time series, including the fluctuations between time steps 50 and 100. Besides, the backcasted detrended production (BDP) based on integration method of Eq. (4.16) is overestimated in comparison with BDP-q^*, because BDP-q^* preserves the original scale of previous period 2009–2015. Panel (b) of Fig. 3 highlights this, and it includes the forecasted detrended production (FDP) and 95% confidence bands for both periods. Considering that upper bands based on Eqs. (3.11) and (3.14) are related to a standardized normal quantile, values of detrended time series cannot be higher than zero; thus, the upper bands are unrealistic. However, the predictions are

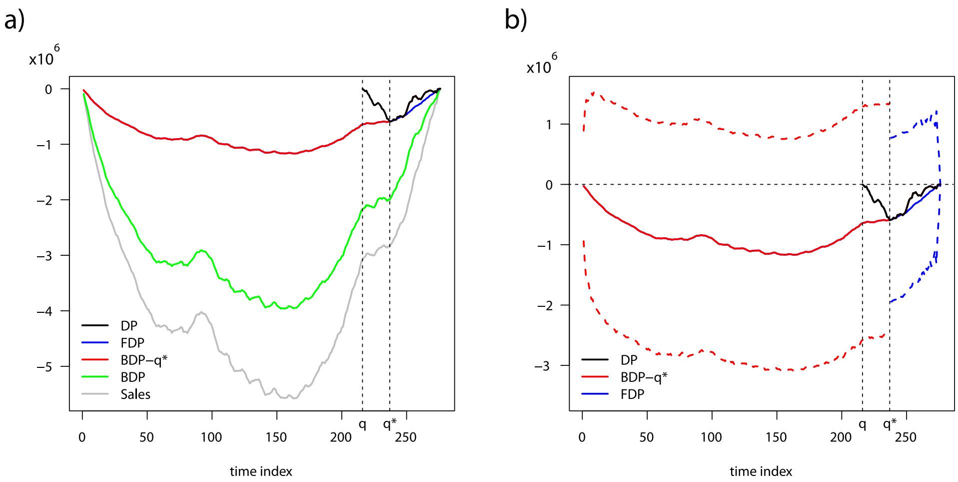

Fig. 3. (a) In addition to time series of Fig. 2a, also included are the detrended production (DP, 2009–2015), the forecasted detrended production (FDP, 2009–2015), using Eq. (3.12), and the backcasted detrended production (BDP-q^*, 1991–2008), using Eq. (3.9) with $q^* = 237$. The value $q = 215$ corresponds to initial time index of DP. The panel (b) corresponds to a zoom of panel (a), but includes a 95% confidence band ($\alpha = 0.05$) for BDP-q^* [dashed red line using Eq. (3.11)] and FDP [dashed blue line using Eq. (3.14)].

influenced by detrended sales time series and DP is located between the 95% confidence bands for period 2009–2015. For steps $k = 1$ and $k = N = 256$, the 95% confidence bands are close to estimations.

5. Conclusions

In this paper the problems of backcasting and forecasting non-stationary time series were addressed. We presented a new approach that updates backcasting procedures based on classical CCF assuming stationary time series, to then obtain non-stationary time series. DCCA provided a fanned estimator for a correlation between two non-stationary time series, and provided then stationary series. Consequently, we obtained a predictor for the transformed time series via CUSUM approach.

A practical example from Chilean cement production time series illustrates the performance of the proposed methodology. Given the difficulty to obtain a long time series for a given period, an extra time series (sales) was used that is highly correlated to incomplete time series (production). The backcasting-DCCA approach allowed to reduce calculus and analysis produced by the cointegration method [12]. Also, given that backcasting and forecasting are based on CUSUM series, the detection of breakpoints based on Page's criterion allowed to find an initial time step (q^*) for these procedures. This could be considered an advantage more than a disadvantage of the CUSUM series beyond the inability to recover original cement time series. Without loss of generality, Page's criterion was considered for forecasting detrended time series and to compare the FDP with DP time series for 2009–2015. However, an earlier time step ($k < q^*$) could be considered.

The proposed methodology could also be considered for environmental and biological non-stationary time series [17,22,23], to estimate missing and/or predict future observations. Moreover, proposed projectors can be developed in further works assuming that both time series share a non-identity covariance matrix [24] or DCCA of two non-stationary time series can be influenced by common external forces [25]. In addition, the presented methodology could be applied in bivariate time-series [24] and to q-dependent detrended cross-correlation coefficient [26] as an extension of ρ_{DCCA} studied here.

CRediT authorship contribution statement

Javier E. Contreras-Reyes: Conceptualization, Formal analysis, Writing - original draft, Writing - review & editing, Visualization. **Byron J. Idrovo-Aguirre:** Supervision, Methodology, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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